Math is fun
HEUREKAS TASKS FOR CHILDREN AGES 7 – 9
All children use and do maths as part of their normal, daily lives! Children are by nature active mathematical researchers. They question mathematical phenomena, discover mathematical principles and are fascinated by numbers and patterns. Children develop playful strategies, for example to win games or count cleverly. For the adults who are part of a child’s growing up process, the biggest challenge is to recognize which situations and actions already contain mathematics and how to utilize this innate potential. That’s because: Math is fun! Children experience this everyday while playing and being curious. They often aren’t even aware of the fact that they are doing mathematics. We hope that this workbook will provide impulses for dealing with mathematics and math situations in a playful, active and creative way. It is in line with the competency model as prescribed in the curricula and framework curricula developed by the various German state education ministries. The workbook offers valuable and practical ways of fostering mathematics competencies at school through use of its unconventional and interesting topics and particular methods. The content can be connected into modules and used in all sorts of different ways in the classroom. The tasks may be done singly as a type of “mini-break” during a lesson, or be used in their entirety for weekly lesson planning, work station learning or for (multidisciplinary) project weeks. We would like to take this opportunity to say a big thank you to the pupils in class 3a at the Christburg-Grundschule (primary school) in Berlin-Prenzlauer Berg. They tested this workbook and its topics at learning stations during a project day in July, 2008 to help us find out how well it works and to assess its suitability for the classroom. Their help was invaluable!

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Hello children,

Math is fun! That’s the name of this workbook that you are holding in your hands. In it, we have put together some games, exercises and activities for you. We hope that you will really have fun doing them and at the same time, see some of the different faces of mathematics. At first, we are going to collect numbers. But not all numbers mean the same thing to all people. That’s something we want to discover together in the chapter called “favourite numbers”. You probably have never thought that giraffes have something to do with maths, but you are in for a big surprise! Ants and even rubbish collection also play a role in this workbook. You will need dice, balls, chairs and a big elastic band. That’s because in this workbook, you are going to stamp things, draw and paint, play Chinese whispers and make up stories! And just imagine, all of these things are: Mathematics!

Have a good time!
To get warmed-up:

Collect numbers and shapes

Make a notebook for your number collection!

Write down how many of the same types of things are found in your classroom. For example: how many coloured pencils are there? How many tables? How many thumb tacks? Are there things that are only found once? Which objects are there a lot of? What numbers can you find in your classroom?

Now go looking for different shapes: Circles, squares, triangles or totally different shapes that maybe don’t even have a name. Draw a picture of the shapes and write down where you found them. If you know, or have made up, names for the shapes you found, write down the names, too.

Write down or draw in your notebook, which numbers you have found and where you found them.

Try to find out as many numbers as you can about your class! Share the results, for example, you could make a wall-newspaper.

Build yourselves a question course

Together, think up questions that you can answer with “yes” or “no”. (For example: Do you like strawberry ice cream?) Write down each question on a piece of paper and place them around your classroom. All the pupils should look at every question. If they can answer a question with “yes,” they should put a building block next to it.

Write down the questions in your notebook. Now work together to decide how you can arrange the blocks so that it is easy to compare how many “yes” answers each question received.
From favourite numbers and friendly calculations

Favourite Numbers

Have you got a favourite number? Why do you like it so much?

Write down your favourite number. Does your favourite number have any friends? Explain to the classmate next to you, how you can tell if numbers are friends!

I always carry my favourite number around with me. Just take a very close look at me. Have you ever seen my number before? Do you know its name? Find out as much as you can about my favourite number!

I know numbers that love each other! 3 and 7 are in love with each other. 1 and 9 are also in love. Can you find a number that your favourite number is in love with?

Write an adventure story about your favourite number and its friends! Paint a picture that shows your favourite number and its friends. Everyone should hang up their pictures in your classroom. Tell each other what’s special about your favourite number and why you like it so much.

There are also numbers that I really don’t like and I call them frog numbers.

Do you know the fairy tale about the frog prince? Once upon a time, there was a frog that had no friends and lived in a dark fountain. But one day, a princess kissed it and the frog turned into a handsome prince. Think up a fairy tale for your own frog number! How can your frog number be turned into a prince number?
Wandering numbers

Now write down each favourite number, its friends and the prince numbers on index cards. Put only one number on each card and you can also write down the name(s) of the pupil it belongs to. Put the cards into different pots (one digit, two digit, and three digit numbers). Now form two to four groups of the same size. The group members should sit in a row one member behind the other. The chairs should be placed with their back rests towards the front.

One pupil rolls the die. When a 1 or 2 comes up, the last pupil in the row takes a card from the one-digit pot, by 3 or 4 from the two-digit pot and by 5 or 6, from the three-digit pot. The pupil then "writes" the number on the back of the pupil sitting in front of them. Just like in the game Chinese whispers, the number wanders forward in this way from one pupil's back to the next. When the number arrives at the front of the row, the first pupil writes down the number on a piece of paper and rings a bell. If the number is correct, the fastest group gets two points. The other groups get one point each if their number is correct. The last pupil in the row moves to the front and all the others move one seat back. (Variation: send the numbers forward one digit at a time.)

Friendly calculations

Write your favourite number, its friends and your prince number on the sides of a blank die. Are some sides still blank? Then think up friends for your prince number and write them down, too. Now your favourite number die is finished!

Now work in pairs. First, make a die with calculation symbols using plus, minus and other symbols that you know.

Then you and your partner throw your favourite number dice and the calculation die to come up with as many calculations as you can. Write down all the calculations on a special page. Now create a solutions page for your calculations so you can tell if your classmates come up with the right solutions.

Which of the exercises that you "threw" do you like most? Mark them with a star or a smiley face. Now exchange your list of calculations with another pair of pupils. Who can correctly solve the most calculations? After all the calculations have been solved, check the other pair's answers.

Are there numbers on your favourite number die that are the sum of the numbers on a classmate’s die? Or what if you subtract - or even multiply - the numbers on two other dice? Do they result in one of your favourite numbers? Go around the room and find at least two classmates that you can make such a calculation and solution with (an equation). Write down the names of the pupils you are able to make an equation with. Now look at the friendly calculations again. Create a “neighbour” calculation for each of them.

A neighbour calculation is made when you make a small change to one or more of the numbers. Now do the neighbour calculations. Are these also friendly calculations if you practice a bit?

Look for “unfriendly calculations” on the calculations pages. Can you make friendly calculations out of the “unfriendly” ones by using neighbour calculations? Now add to your picture gallery. Draw a picture that shows one or more friendly calculation.

Editors note: Blank, wooden dice that can be written on are available at low cost from mail order companies for school supplies.
Creating Patterns

Try to draw the pattern or spots giraffe’s have! Which shapes did you use? Round or rectangular? Triangles? Squares? Or totally different shapes?

Which forms did your classmates use? Hang up your giraffe pictures (either on the wall or with magnets on the board) in your classroom! Look at the patterns your classmates drew. Decide together which pattern looks the most like the one on a real giraffe. Cut out forms for a giraffe pattern from paper. First place the forms on your blank giraffe, and then put the forms on the overhead-projector. Take a photograph of the overhead-projector picture.

What shapes do the spots have? How close to each other are the spots?

How can you tell if a pattern is a giraffe pattern? Work together to describe this as exactly as you can.
Giraffe Stamp

Make a stamp out of strong (corrugated) cardboard (and cork for the handle) that you can use to stamp your giraffe pattern: triangles, rectangles, hexagons, stars, round forms, circles and anything else you think of. Now start stamping your giraffe pattern.

Experiment with your stamping! Exchange stamps with each other! Which looks better: giraffe patterns made using only one or two different stamps, or those that were made using lots of different stamps?

Fantasy Giraffes

Work with a partner and write the directions for how to draw or print a good giraffe pattern. Give your directions to another class and let them test how well your instructions work!

Look at pictures of giraffes in books or on the internet. There are different kinds of giraffes. Try to recreate their patterns by using paper cut-outs, drawing or stamping.

Can you find suitable names for the fantasy giraffes you created?

Draw or stamp a giraffe pattern that looks as much as possible like a real giraffe pattern. You could even make a giraffe t-shirt. All you need is a yellow t-shirt, brown fabric paint, and a paint brush. Then you can get started!

Fantasy Giraffes

Do you know of another animal that has a special pattern? Can you draw or stamp this pattern?

Zoo Trip

Take a trip to the zoo and take photographs of animals there that have nice patterns. (Or, you can look at pictures in books or on the internet.) Paint pictures of the animals to create a zoo gallery in your classroom. Compare the patterns: which shapes do they have? Which are similar and which ones are completely different?
Have you ever thought about how the driver of a refuse collection lorry knows which route they have to take? Plan your own refuse collection lorry route!

The lorry’s route should be on this street plan and must include every street at least once but avoid using the same streets twice as much as possible. Where must the lorry start so that no street is used twice?

Normally, the refuse collection lorry must return to its starting point. Is this possible without having to drive back using the same route?
Imagine that your pen is the refuse collection lorry. Can your lorry pen drive on these figures without having to drive any way twice? With which figures does your pen end up at your starting point after you have finished “driving”?

Why can’t you always complete a figure in one flow without having to pick up your pen or draw double lines? Look carefully at the places where your pen “got stuck”. How many streets intersect at these places?

Look at a map of where you live. Are there lots of streets there? What kinds of intersections are there? Are there intersections that a rubbish collection lorry has to use twice? Have you ever seen this happen?

Think up a difficult rubbish collection route and then draw it on a piece of paper. Then draw it again on the playground so that it is big enough to dribble a ball on the streets (or drive on it with a model rubbish collection lorry). Now pupils from your class should each choose a street and then stand on it.

Then one child starts dribbling the ball through the streets. When the ball goes past a pupil, that child must sit down. When all the children are sitting down on the streets, the rubbish collection route has been driven completely.
Patterns for the ants
Kolam pictures from India

Early in the morning in the south of India, the women draw pictures out of rice flour on the freshly cleaned floors in front of their houses. These Kolam pictures are supposed to invite the god of prosperity to come into their houses. They are also supposed to remind people that humans aren’t alone on the Earth. Millions of other creatures, for example tiny ants, live here too. The rice flour used for the pictures is also food for the ants and other small animals.

Drawing game
Choose one of the tasks from A – E. Then read the task and take one of the index cards with one of the shapes you drew. Now study and practice the task and finally, find someone who will do the task you chose.

**TASK A**  
Place five chairs in the given 1-3-1 pattern. Blindfold your partner’s eyes and then lead them in the given pattern around the chairs. Lead your partner through it as many times as necessary until they know the pattern well. Then your partner should walk the pattern with their eyes open. Finally, your partner should draw the pattern.

**VARIATIONS**

1) All the other pupils should stand in a circle around the pattern. Lead your partner by asking the child standing closest to the direction where your partner should go next to make a noise. For this to work, all the children must have seen the form or you have to always point to the child who should make the next noise.

2) Lead your partner by making noises yourself that they should follow.

3) Lead your partner very slowly by holding something scented in front of their nose that they should follow. For example, spray perfume on a handkerchief.

**TASK B**  
Place five pupils in the given 1-3-1 pattern. Give your partner an elastic band or a long piece of rope tied in a circle. Use only words to tell your partner how to place loops around their classmates in order to keep the pattern in place. Afterwards, the pupils go out of the band or rope and should draw the pattern.
Points are often drawn on the floor. Then a line is
drawn that winds around the points. On these two
pages there are several examples with 1-3-1 points.

**TASK C**  Take a piece of thick construction paper that is about 30 cm x 42 cm in
size. Mark the 1-3-1 pattern using coloured stones. Take two board magnets
and give one to your partner and place the other one on the paper. At least two of your
classmates must lift up the paper so that you and your partner can reach beneath it. Guide
your partner's hand that is holding the magnet under the paper so that the magnet tra-
ces the pattern. When your partner is sure about the pattern, they should draw it.

**TASK D**  Draw the 1-3-1 pattern on a large (circa 15 cm x 21
cm) index card. Trace the pattern with a glue stick
and then sprinkle the card with enough sand so that
you can easily feel the pattern. Hang up the card
using five thumb tacks that you stick through the 1-
3-1 pattern. Blindfold your partner and guide them
to the pattern. Let them touch it with both hands.
Now take down the card and have your partner draw
the pattern.

**TASK E**  Draw the pattern on your
partner's back with your fin-
ger. Remember the 1-3-1 points. Your part-
ner should draw the form at the same time
on a piece of paper.

**VARIATIONS**  Trace the pattern with
your finger in your part-
ner's hand. They should then draw the pattern.
Or guide your partner's foot along the pattern.
**Brahma’s knots**

Here is an example of a famous Kolam figure. It is called Brahma’s knot and is drawn using 1-3-5-7-5-3-1 points. Can you draw it on a piece of paper or walk it after you mark the points on the floor?

What does Brahma’s knot look like if it has 1-3-5-3-1 points? And what about with 1-3-5-7-9-7-5-3-1 points? Form groups of four and try this out either on the board, with chairs and rope or on the playground with chalk. Or you can work together with paper and pen. Brahma’s knot should be colourful. The fields formed by the line should have a different colour. How many colours do you need?

**Fish pattern**

Think up your own Kolam pattern! Always draw the points first, then draw a few, or even just one winding line around the points to create the pattern. Try to make a pretty, even pattern.
Comments, methodical suggestions and solutions

Dear Teachers,

We would like to provide you here with some comments about the exercises and give you a few suggestions about methods of using them. We hope that our ideas will help you in your efforts to prepare and teach mathematics lessons that are creative and encourage your pupils to research and discover. Above all, our goal is to assist you in showing your pupils that: Math is fun! By doing so, you are also helping them to believe the motto of the Year of Maths which is: “You are better at maths than you think you are!”

Collecting numbers and shapes

The purpose here is to increase pupils’ perceptions and thereby, liven up numbers and shapes. A positive side effect is that pupils also get their first experience with statistics. The presentation of the results is very important for the step of pupils being able to visualize and formulate their collected data. This can be done either through wall posters, as a “market” where children exhibit their results at “stands”, or through individual presentations. An idea that makes sense is to use the “think-pair-share” method in which the pupils first work five minutes alone. Then they spend ten minutes going around the room whispering their results to each other. Here, each of them passes on only one number or shape per contact. At the end, the results are given in a joint plenary presentation and, for example, exhibited on wall posters.

From the questions with the results that can be presented through numbers of building blocks, a presentation using block diagrams can easily be prepared. A condition here, of course, is that the pupils have come up with the idea that this topic can be presented more clearly if they put the building blocks into stacks. (See Micahela Naumann: “Meine Klasse in Zahlen”, pp. 16-19 in: Grundschulunterricht 2-2008, Mathematik: Daten - Zufall und Wahrscheinlichkeit – Kombinatorik.)

Another idea: re-use the building blocks for statistics found in your text book or for football charts or statistics from the pupils’ families. With the latter, it is important to make sure the data is kept anonymous and point out the importance of data protection and privacy.

Favourite numbers and friendly calculations

The pupils have to agree what exactly friends of numbers should be. They can also choose the “numbers who are in love”. That’s why the tip from Heureka is there. Then the pupils must decide which rule to use when changing “in love” numbers to multiple-digit numbers. Class 3a from the Christburg-Grundschule made the following decisions: “In love” numbers are two single-digit numbers that add up to ten. With multiple-digit numbers, there were “ones love”, “tens love”, and “hundreds love”, etc. For example, the “ones love” from 43 is 7; the “tens love” from 43 is 6. There are many other possibilities.

“Almost in love” numbers can also be useful. The “almost in love” of 6, for example is 3 and these add up to 9. You need “almost in loves” when, for example doing complementary calculations for 100. For example: Find the complementary number of 234 to 1000! Here you need the almost in love hundreds number 7, the almost in love tens number 6 and the ones in love number 6. The solution: 766.

Through use of favourite number as a starting point, the pupils should obtain an access to numbers that is personally identified. (See Urs Ruf, Peter Gallin: “Ich mache das so! Wie machst du es? Das machen wir ab. Sprache und Mathematik 1. - 3. Schuljahr”, Lehrmittelverlag des Kantons Zürich 1995). The pupils can formulate their likes and dislikes about individual numbers and then work further with these numbers.

Calculations with favourite numbers, their friends and the frog numbers help pupils find their individual approaches to numbers and calculations.

“Friendly calculations” help pupils feel more comfortable with unusual or seemingly complicated calculations. Often, the “unfriendly calculations” really aren’t that far away from “friendly calculations”. Additionally, this task fosters communication and argumentation skills, as these competencies are needed to explain why a calculation is either friendly or unfriendly. Every child has their own personal preferences and therefore, each child must also find their own arguments.
Giraffe patterns

A nice way to begin this topic is to display the giraffe outline on the overhead projector. Then sprinkle a handful of confetti or star shapes on it. Then ask "is this is what a giraffe looks like. No? What does a giraffe look like?" The overhead projector can also be used later (in phases of individual or group work) for experimenting. The advantage is that the projected giraffe image on the wall is much bigger than the one on the pupils' papers and therefore, the effect of the pattern is clearer and has greater impact. It can be difficult to give names to the different shapes of the giraffe spots as these could have irregular, straight or crooked borders or have re-entrant corners or frayed edges. The exercises in chapter 1 (Collect numbers and shapes) works to prepare for this. These can also be used as part of the giraffe pattern exercises, for example as a task in an open learning environment. The giraffe pattern task is particularly suited for working with open methods (learning stations, projects, learning diary and so on) as exploring and inventing are in the foreground.

Inventing a true-to-life giraffe pattern requires careful examination and the development of improvements during several stages until the result comes close to the pupil's individual concept.

The fact that every giraffe will look different makes possible individual solutions and solution paths and opens up a creative "field of research".

In other words, recognizing what the differences are between the various results trains dealing with mathematical terms and analytic skills. The giraffe pattern topic could lead to work with tessellation, which, unfortunately, we did not have room for in this workbook. The giraffe's spots are irregular tessellations on a plane (at least on reticulated or "net" giraffes). From this starting point, a discussion could then be held about what pretty bathroom tiles look like. Because one needs a lot of them, they should all have the same shape. Which shape is most suitable? How can one, for example, create an interesting form using a square? This is simple: you cut a bit out of one side and place it on the opposite side. This can be repeated in order to create tiles that even have a fractal effect. Use a photo copier to duplicate the sample tile as often as desired, cut out the tiles, paint them (if desired) and glue them. M. C. Escher has suggestions for interesting tiling that has aesthetic value under: http://www.fraktalwelt.de/escher/elinks.htm.

Driving the refuse collector lorry

A good refuse collection route is one in which no part has to be driven twice and there are no dead ends or intersections where an uneven number of streets cross. Children can come to this conclusion by testing and creating their own street maps. Tying this out on large maps, either drawn or glued to the floor, is motivating and requires cooperative strategies. In this way, the problems involved can be experienced directly. Quite a bit of abstractive ability is needed in order to transfer the route onto paper.

Solutions: Street map: Beginning and/or end at one of the two three-way intersections. The round trip only works with a double trip on the route section between the two three-way intersections. With A, E and H, the route ends where it began. D and F have different start and end points. The rest of the figures cannot be "driven" in one flow without drawing double lines. For further reading about this see: Stephan Hußmann and Brigitte Lutz-Westphal: "Kombinatorische Optimierung erleben", Vieweg 2007, Chapter 3. See also page 16.

Patterns for ants

The ethnological background encourages questioning and investigating. An interesting aspect here is that even though this is a more female-oriented topic, it has equal appeal for both genders. The art of drawing Kolams is traditionally passed down from mother to daughter. Similar to mandalas, the Kolam patterns are full of symmetry and harmony. They offer the rare opportunity to talk about mathematical objects that are based on an activity that is performed solely by females. Here, pupils could profit from a multidisciplinary lesson that combines maths, art, religion, ethics and general sciences.

See also: http://www.academicinfo.net/mathethno.html

Our goal here is that pupils use all of their senses to experience these very aesthetically pleasing ornaments. Combining movement in the framework of a communicative task fosters communication skills and provides basic experience with space and shape. The various tasks can, of course, also be used for more difficult or self-created patterns. A characteristic of Kolams is that they can be formed with a single line. This is what makes it possible to create them using the elastic band.
The task of finding the 1-3-5-3-1 points in Brahma’s knot is quite challenging. Through the depicted figure based on 1-3-5-7-5-3-1 points, the guidelines are very concrete and it is a challenge (and a problem solving exercise) to transfer it to the smaller figure. Finding reasons for the invented figure requires argumentative abilities and communication skills. Here is one solution. However, it is entirely possible that the pupils will come up with another, equally logically-based solution:

Another technique for creating and tracing Kolams is to first put them together using little pieces and then later try and find a path using the closed line. For example, using the knot based on the 1-3-5-7-5-3-1 points, one can first draw the 1-3 points for the upper and under “blossom” and then divide the middle part into smaller, closed curves. Videos (for example on YouTube) demonstrate that many Kolams are actually drawn piece for piece like this on the ground. In this way, the basic elements of the Kolam figure can be found and then used as “building elements” to create new patterns. Here, as well, problem solving strategies are being fostered. For example, “How do I deal with a very complex task?” This task trains the strategy of not looking at the big picture for a moment, but rather, trying to find a solution in a smaller, limited area and then applying this partial solution to the whole. An advantage here is that it is quite easy to remember the piece-by-piece drawn Kolams. If time is short during the lesson, it should be decided whether to put more emphasis on the swinging, curvy, drawing method that uses movement in one flow or the easier to remember method of dividing the figure into sections that repeat.

The painting of the Kolam figures can lead to a continuation of the topic by dealing with the well-known four colour problem. It is interesting that because they consist of one closed line, Kolam figures can be painted using only two colours. This means that butting surfaces can be differentiated (along the line) through use of two colours. This can be demonstrated easily on the overhead projector by first placing a blank piece of coloured paper on the screen (one colour). Then a rubber band circle is placed on the screen. This represents the line and creates a separate area (second colour). This band can either be looped into a pattern, or other rubber bands can be added in loops to create the pattern. This process can be repeated as often as desired, but only two colours are used: one for the original area and one for the areas within the rubber band loops.
How does PRIMAS achieve this?
PRIMAS supports teachers in integrating and applying inquiry-based learning pedagogies in their mathematics and science classrooms. During the project’s lifetime (2010-2013), various resources and support measures will be developed and made available to teachers, parents and pupils. PRIMAS offers support for professional development facilitators, professional development courses for teachers, teaching materials and support for teachers, as well as information meetings for parents and pupils. Furthermore, PRIMAS works in close cooperation with school authorities and other key institutions in national education systems across the partnership. This helps to ensure the best possible conditions for the integration and application of inquiry-based learning in classrooms.

What does PRIMAS have to offer?
For teachers: PRIMAS offers a broad range of teaching materials for inquiry-based learning in maths and science, both for primary and secondary education. PRIMAS also offers professional development courses in IBL pedagogies as well as shorter events for teachers.
For parents and pupils: In IBL, pupils can independently try out different strategies to solve problems and do research, experiment and discover on their own. This makes it possible to gain a deeper understanding of important concepts, learn problem-solving skills, experience new approaches to solving science and math problems and also develop effective team work skills. PRIMAS also works closely with parents and pupils, for example through a range of special events on IBL for parents.